Equilibrium Equation for Spatial Frame Structure Constructed in Stages and Its Applications

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Summary

For a spatial frame structure constructed in stages, the unstressed state of an element is defined as its initial geometric shape with zero internal stress. Using energy method, a new equilibrium equation for spatial frame structure constructed in stages is established. In this paper, a simple spatial truss is used as an example to illustrate the application of this new equilibrium equation. Calculation demonstrates that the final member forces and structural displacements can be uniquely determined for a structure constructed in stages with any given structure type, loading and boundary conditions and without knowing the actual construction processes and methods, as long as the unstressed state of all structure modules or components remain unchanged. And the final internal forces and structural displacements are irrelevant to construction processes.

Keywords: One-time built structure; Structure constructed in stages; Unstressed state; Spatial frame structure; Construction control

1. Introduction

Traditionally, for small-scale structure, one-time built construction method was usually used with a shoring system to support its weight during construction. We can use the Eq. (1) to obtain the final nodal displacements and internal forces $[K]\{\delta\}=\{P\}$ (1). ($[K]$ = global stiffness; $\{\delta\}$ = nodal displacement). However, as span and dimensions increase, this method becomes unsuitable because of equipment and environment issue. Thus the method of construction by stages is developed. For structures constructed by stages, each member's weight is added to a system-changing structure step by step following certain sequence. For each stage, structural system is changing, the increment could be acquired by $[K_i']\{\Delta\delta_i\}=\{\Delta P_i\}$ (2) ($[K_i']$ = global stiffness at certain construction stage). The final displacements and internal forces can be obtained by summing up all the increments.

However, this method has several problems. (1) There may be tens of construction stages for large-scale structure, thus the calculation is complicated and large in amount; (2) Effects of temporary loads, such as temperature, should be considered into each construction stage, making analysis more difficult; (3) Only one operation can be considered at each stage, making parallel operation of multiple working procedures impossible, which is important to reduce construction time.

In this paper, we include geometrical parameters - 'unstressed state' to establish direct connection between final state and every construction stage. Meanwhile, the effect of each operation is

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substantially distinguished, making parallel operations easy to be realized.

2. Equilibrium Equation of Structures Constructed by Stages

Element's unstressed state is defined as its state with zero internal stresses. For space beam in Fig. 1, the unstressed state include unstressed length l_0, unstressed bending curvature at two ends K_{yi0}, K_{yj0}, K_{zi0}, K_{zj0} and unstressed torsion angle φ_0.

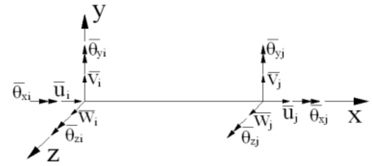


Fig. 1: Space Beam Element

For balanced structure: $\delta\Pi = \delta U_{\text{total}} - \delta W = 0$ (3)

For each element, strain energy is $U_{\text{total}} = U_{\text{axial}} + U_{\text{bendz}} + U_{\text{bendy}} + U_{\text{torsion}}$.

In which, $U_{axis} = \frac{EA}{2l} \cdot \left(u_j - u_i + l - l_0\right)^2$; $U_{\text{torsion}} = \frac{GI_p}{2l}\left(\varphi - \varphi_0\right)^2$;

$$U_{bend,z} = \frac{EI_z l}{6}\left(\Delta K_{zi}^2 + \Delta K_{zi}\Delta K_{zj} + \Delta K_{zj}^2\right); \quad W = \sum_{k=1}^{l} P_k \cdot \Delta_k .$$

Substituting them into Eq. (3), we get a matrix equation $[K]\{\delta\} = \{P\} + \{P_0\}$ (4)

Eq. (4) is exactly the equilibrium equation of space frame structure constructed in stages.

In Eq. (4), $[K]$ is identical to that in Eq. (1); P_0 is an additional load representing the difference of elements' unstressed states between structure constructed in stages and one-time built structure. For the latter, P_0 becomes 0 and Eq. (4) is simplified to Eq. (1). Through this new equilibrium equation, we can directly determine final state without any consideration of stages. Accumulation is no more necessary.

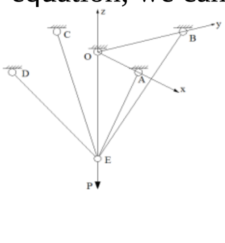
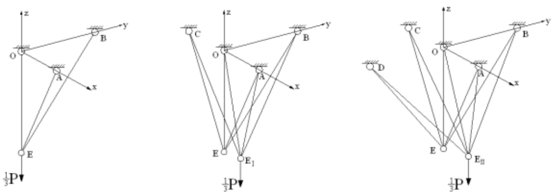


Fig. 2: One-time Built Truss


(a)Stage I (b)Stage II (c)Stage III
Fig. 3: Truss Constructed in Stages

3. Example

For spatial truss in Fig. 2, final state can be obtained by Eq. (1). Assuming it was constructed by three stages shown in Fig. 3. The nodal load P was also added by three stages, with an increment $P/3$. We acquire final state separately by two methods: traditional incremental accumulation and direct calculation using the new equilibrium equation in this paper. Calculation shows that final displacements and unstressed state of two methods are the same, while different to that of one-time built structure.

Comparing calculation result of structure in Fig. 2 and Fig. 3, we can see that two completed structures are actually not identical even with totally the same final structural system, external loads and boundary conditions because of unstressed length difference of EC and ED. With an in-depth study, we can see that for structures constructed in stages, structural loads are actually assembled onto a deformed structure, leading to the difference of unstressed state, and thus generalized an additional load term P_0 .

In construction of cable-stayed bridge, cable force adjusting is actually adjustment of unstressed length, and the elongation is exactly unstressed length variation. Therefore, unstressed length can be adjusted as controlling parameter to achieve the predefined unstressed state, and then the designed structure can be obtained. This method, already applied on tens of major bridges in China (Huang 2003), has significantly simplified calculations of construction stages and reduced site work.

4. Conclusions

(1) The difference between one-time built structure and the final state of structure constructed in stages is actually the difference of their unstressed states of each member.

(2) New equilibrium equation in this paper can be used instead of incremental accumulation to directly determine final state of structure constructed by stages with no consideration of stages.

(3) With given external loads, structural system and supporting boundary conditions, only if all structural members' unstressed states are predefined, the final structural deformations and internal forces could be uniquely determined and irrelevant to construction processes.